

How to Push a Block Along a Wall

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ABSTRACT

Some robot tasks require manipulation of objects that may be touching other fixed objects. The effects of friction and kinematic constraint must be anticipated, and may even be exploited to accomplish the task. This paper analyzes an example task, presents a dynamic analysis, and derives appropriate effector motions. The goal is to move a rectangular block along a wall, so that one side of the block maintains contact with the wall. We construct two solutions that push the block along the wall.

1. Introduction

Consider the problem of pushing a rectangular block along a wall, so that one edge remains in contact with the wall. A few experiments (try pushing a paper-clip box with a paper-clip) will yield two solution strategies, and will also yield a variety of failure modes (Figure 1). This paper derives the two solutions from a dynamic analysis, and shows that there are no other solutions. The approach is to derive the entire mapping from applied force to block motion, and to compare this mapping with the set of all forces that can be applied through a pushing operation. In the process, we demonstrate the analysis of multiple-contact friction dynamics including distributed support friction, using acceleration centers to represent force, as described by Brost and Mason (1989).

1.1. Background

This paper falls in an area that has attracted considerable attention: rigid body mechanics applied to manipulation. The seminal work in this area is Simunovic's (1975) analysis of peg insertion, which was further elaborated by Whitney (1982). Ohwovoriole, Hill, and Roth (1980) provided a more general treatment, which was later extended to three dimensions (Ohwovoriole and Roth 1981). This line of work is mainly quasi-static: inertial forces are assumed negligible, motions are inferred from the direction of any imbalance of static forces. Later work, primarily Erdmann (1984) and Rajan, Burridge, and Schwartz (1987), included dynamic forces and uncovered some interesting subtleties, such as the existence of ambiguities, where the motion of the object may be under-determined. The present paper applies the methods of Erdmann, and Rajan et al., but uses the graphical representation of force described by Brost and Mason (1989).

Related work in the mechanics of grasping is also relevant to the present paper. In particular, we are constructing the locus of contact forces that can be applied on the perimeter of an object, as described by Mishra, Schwartz and Sharir (1986).

An important element in the present paper is the presence of frictional forces that are distributed over a positive area, rather than at known discrete points. Our estimates of the resulting forces draw primarily on (Mason 1986). Better estimates can sometimes be obtained: see (Peshkin and Sanderson 1988; Goyal 1989).

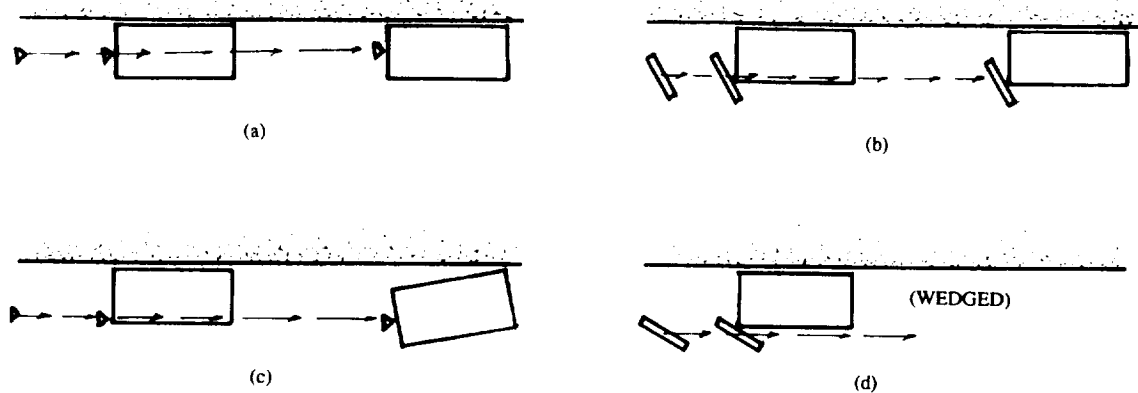


Figure 1: Four different pushing motions. (a) and (b) work just fine. (c) causes the block to lose edge-to-wall contact. (d) is wedged—the block will not move no matter how large the forces.

2. Representation of force by acceleration centers

This section reviews the representation of force by acceleration centers. This approach, as described by Brost and Mason (1989), yields a graphical method to analyze planar contact problems. The construction is similar to the use of velocity centers to analyze kinematic constraints, described by Reuleaux (1876). The key observation is that the velocity of any plane body can be described as a rotation about some motionless point called *velocity center*, or *instantaneous rotation center*. Obviously this would not work for a purely translational motion, but these can be handled by allowing the velocity center to range over the *projective plane*—a translation gives a velocity center at infinity. The velocity center is easily constructed: construct two lines, each orthogonal to the velocity of some point on the body. The intersection of the two lines is the velocity center.

The definition of an acceleration center is similar. For any plane acceleration, there is an unaccelerated point, called the *acceleration center*, which ranges over the projective plane. We can use the acceleration center to represent forces. Given some plane body with a mass m and angular inertia I , we can map any plane force into the resultant acceleration center. This mapping has a very useful property—the magnitude of the acceleration, and hence the magnitude of the force, is not represented. For problems involving frictional contact, this property is very useful, because the magnitudes of the contact forces are not constrained, only the lines along which the forces act.

In practice the use of acceleration centers to represent applied forces is quite simple. Some examples are shown in Figure 2. We place the center of mass at the origin, and choose a unit distance equal to the radius of gyration. Then the acceleration center lies on a perpendicular to the force through the origin. The acceleration center's distance from the origin is the inverse of the force's distance from the origin. Because it is necessary to represent the sign of the moment of force, we will use two projective planes with a common line at infinity. One plane corresponds to positive moments, one plane corresponds to negative moments, and we have the line at infinity for purely translational accelerations, i.e. zero moments. Topologically, the space is equivalent to a sphere. The upper hemisphere corresponds to positive moments, the lower hemisphere to negative moments, and the equator to zero moments.

The properties and applications of this mapping are more fully described by Brost and Mason (1989). We note two key properties:

- The mapping is nearly dual: a directed line of force maps into a point, and a point, corresponding to the set of all forces passing through that point, maps into a line, the locus of acceleration centers.
- Positive linear combinations of forces map into convex combinations of acceleration centers.

These properties are ideal for representing frictional contacts:

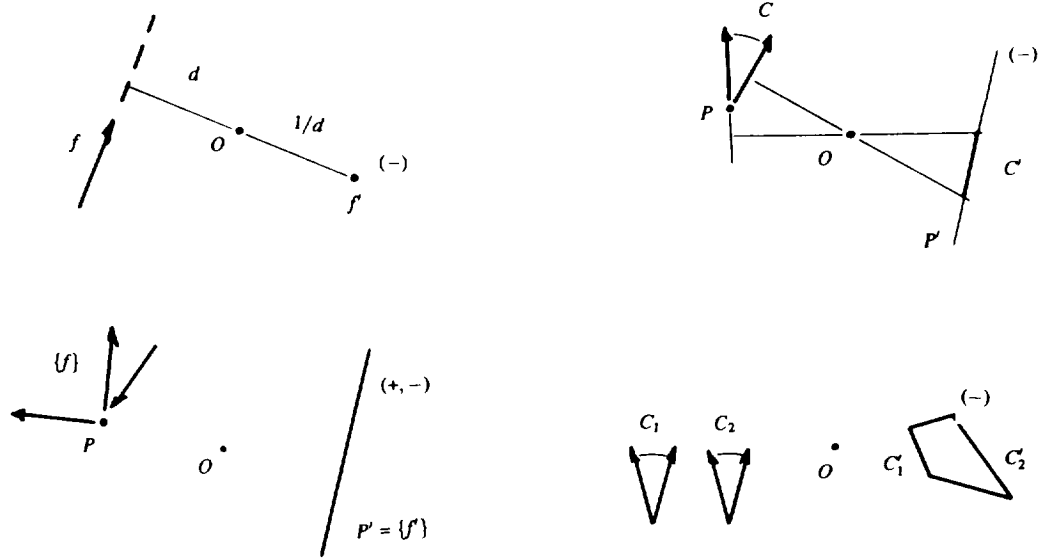


Figure 2: Some example acceleration centers, illustrating properties of the mapping of force to acceleration center. Here we have super-imposed the plane of positive moments and the plane of negative moments, using (+) and (-) to distinguish the points.

- The feasible motions at a point contact are represented by a linear constraint on the feasible acceleration centers.
- The feasible forces at a point contact are represented by a line segment, which is the locus of acceleration centers corresponding to a friction cone.
- The resultant of several friction cones, arising from several simultaneous contacts, lies in the positive linear combination of the forces, which defines a convex polygon in the space of acceleration centers.

Geometrically, the only real inconvenience is that we have two planes and a line at infinity. Sometimes we draw the planes separately; sometimes we superimpose them. When two points must be joined by a line segment, the construction is sometimes counter-intuitive: with the two planes superimposed, draw a line through the two points. Now, if the two points are on the same plane, the line segment is the part of the line between the two points, as usual. But if the two points are on different planes, take the part of the line *outside* the two points, and also include a point at infinity. The method will be illustrated by example.

3. The block-along-wall problem

The block-along-wall problem is formulated as follows.

- A rigid rectangular body is free to move in the plane, with one edge initially against a straight wall.
- We assume Newton's laws with Coulomb friction. Gravity acts normal to the support plane. Friction occurs with the wall and with the support plane. The distribution of support forces is unknown, and may vary with time.
- The goal is to move the object forward while keeping one edge against the wall.

To analyze the operation, we enumerate the contact modes, and determine necessary applied forces for each mode. A peculiarity of rigid body mechanics is that some forces are consistent with more than one mode. Nonetheless, a particular contact mode, i.e. sliding along the wall, can be assured by applying a force consistent with the desired

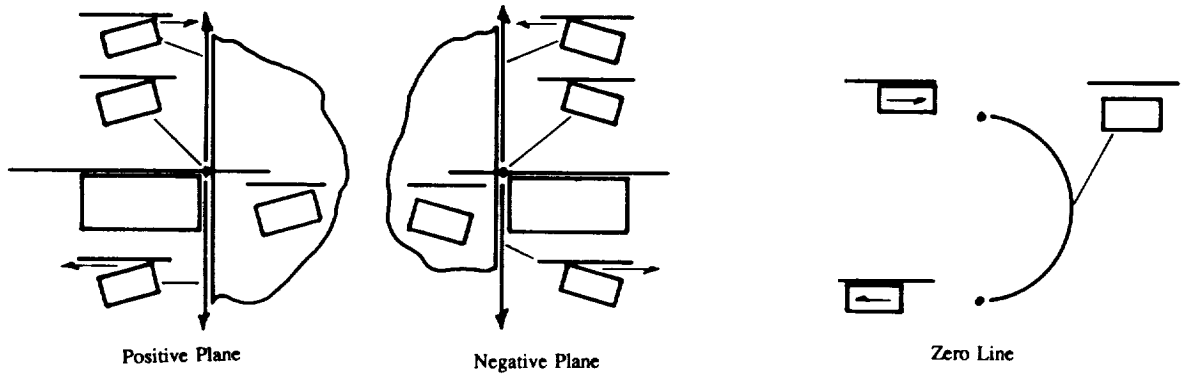


Figure 3: Acceleration centers A for each contact mode. We draw the block tipped to indicate which contacts are broken, and arrows show relative motion at any remaining contacts. The zero line is drawn as if the positive plane and negative plane were projected onto the northern and southern hemisphere, respectively, of a sphere. Then the zero line is the equator, as it would appear from the north pole. There is one contact mode not shown—rest—which corresponds to zero acceleration.

mode and inconsistent with any other modes. This observation allows us to specify constraints on applied force to produce the desired motion.

Our analysis will proceed as follows:

1. Enumerate the contact modes $\{i\}$.
2. For each contact mode,
 - (a) Construct acceleration forces A_i ,
 - (b) Construct wall contact forces W_i ,
 - (c) Construct support friction forces S_i ,
 - (d) Construct pushing forces $F_i = A_i \ominus W_i \ominus S_i$.

where $X \ominus Y$ is the set of all forces $x - y$, for $x \in X$ and $y \in Y$.

The result is a mapping from each contact mode i to a set of applied forces F_i . Some of these applied force sets overlap, so that the contact mode is not always uniquely determined by the applied force.

The first step is to enumerate the contact modes. Figure 3 applies Reuleaux' (1876) partitioning of the space of motion centers to determine the set of feasible contact modes. At each kinematic constraint, we construct a contact normal. To the right (left) of the normal only positive (negative) rotations are feasible. On the normal itself, either direction is feasible. We also construct the contact tangent—above the tangent positive rotations cause rightward motions and negative rotations cause leftward motions. Below the tangent, the opposite is true. The two contacts give rise to two normals, and a single tangent, which cut the space of acceleration centers into different sectors. Each sector, and each boundary segment, potentially corresponds to a different contact mode.

The next step is to iterate through every contact mode, constructing the corresponding set of applied forces. We illustrate the procedure for only one contact mode, namely the desired mode which pushes the block to the right.

(a) Construct acceleration centers A .

Figure 3 shows the acceleration centers for each contact mode. There is only one acceleration center for the desired mode, rightward sliding, which is on the line at infinity.

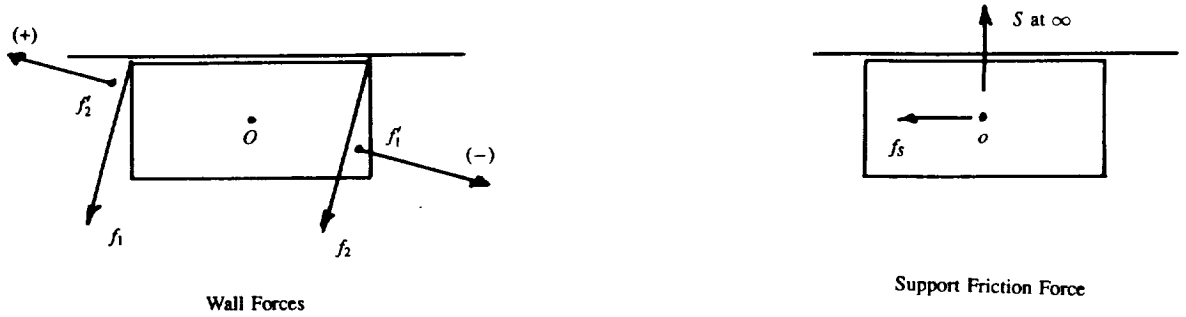


Figure 4: Wall contact forces W , and support frictional forces S for the desired motion.

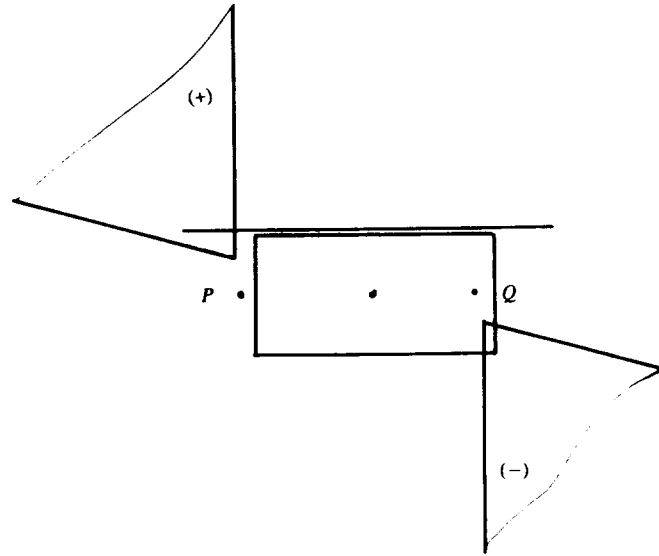


Figure 5: Applied forces F for the desired motion. The applied force must give an acceleration center in the indicated regions. An equivalent constraint is that the applied force must pass between P and Q , and make an angle greater than $\tan^{-1} \mu$ with the wall normal.

(b) Construct wall contact forces W .

The possible wall forces can be represented by two point contacts, one at each corner of the block. For the desired contact mode, rightward sliding, Coulomb's law constrains the direction of each force as shown in Figure 4. We construct acceleration centers for each force, and form the convex combination, to obtain the line segment shown.

(c) Construct support frictional forces S .

For the desired contact mode, rightward sliding, the support frictional force reduces to a single force acting through the center of mass, as shown in Figure 4. This maps to an acceleration center at infinity.

(d) Construct applied forces $F = A \ominus W \ominus S$.

The set of forces $X \ominus Y$ is the positive linear combination of X with $\ominus Y$, so we simply take the convex combination of the positive plane of X with the negative plane Y , and the convex combination of the negative plane of X with the positive plane of Y . When we apply this procedure to compute $A \ominus W \ominus S$, we obtain Figure 5.

3.1. A rotating contact mode

The main complication is in constructing the set of support frictional forces. For the desired contact mode it was easy, because a pure translation was involved. For rotations, the set of support frictional forces is partially indeterminate.

We can, however, construct a set S that represents a bound on the support frictional force. Figure 6 illustrates the method for a contact mode that involves rotation of the block. A bound on S is obtained by applying two constraints:

- A positive (negative) rotation requires a negative (positive) moment with respect to the center of mass. A translation requires zero moment (Mason 1986).
- If the support region lies in some sector with respect to the rotation center, then the force lies in a similar sector, rotated ninety degrees.

These constraints are sufficient to support synthesis of a block-pushing strategy. For other applications, more detailed approximations are required (see Peshkin and Sanderson (1988) for example).

By repeating the procedure for all contact modes, we obtain the complete atlas of Figure 7. This atlas is a multiple-valued mapping from applied force to contact mode. This mapping is an approximation; for some applied forces, some of the predicted motions cannot really occur. But any motion that can occur will be included in the predictions. Hence, where a unique contact mode is predicted, that prediction is a correct one. Note that the desired contact mode, rightward sliding, does not overlap other modes. We can guarantee the desired motion by generating any force in the region. The problem of generating the required force is the subject of the next section.

4. Synthesizing a pushing motion

When we push the block, an additional frictional contact is applied somewhere on the boundary of the block. The problem is to determine where to push, and in what direction. First we construct the set of all forces that could be generated by pushing the block (Figure 8). Then, by intersecting with Figure 5, we identify two classes of valid strategies (Figure 9). To produce the desired motion, either push with a left-sliding contact along the trailing edge of the block, or use the face of the finger to push at the trailing vertex of the block.

5. Discussion and Summary

The motion center approach is well-suited to problems involving planar frictional contacts. For the support distribution, however, this approach has some limitations. The main limitation is that the motion center approach does not represent magnitudes, and the support friction is bounded in magnitude, unlike constraints in the plane. For example, if we naively construct the set of applied forces that can lead to rest, we obtain the set of all forces. In truth, a *small enough* force in any direction leaves the object at rest, but a large force in the same direction will accelerate the object. For the sliding block problem, we can manage this deficiency, but it represents a general difficulty for which there is no obvious remedy.

A second problem, which is not particular to the motion center approach, is the indeterminacy of the support distribution. The main difficulty in problems of this kind is to find some characterization of the support distribution that leads to a useful characterization of the motion. This paper assumes a known centroid, with the support confined to a known rectangle, which happens not to include feasible rotation centers. I am working to extend the method to more general support distributions.

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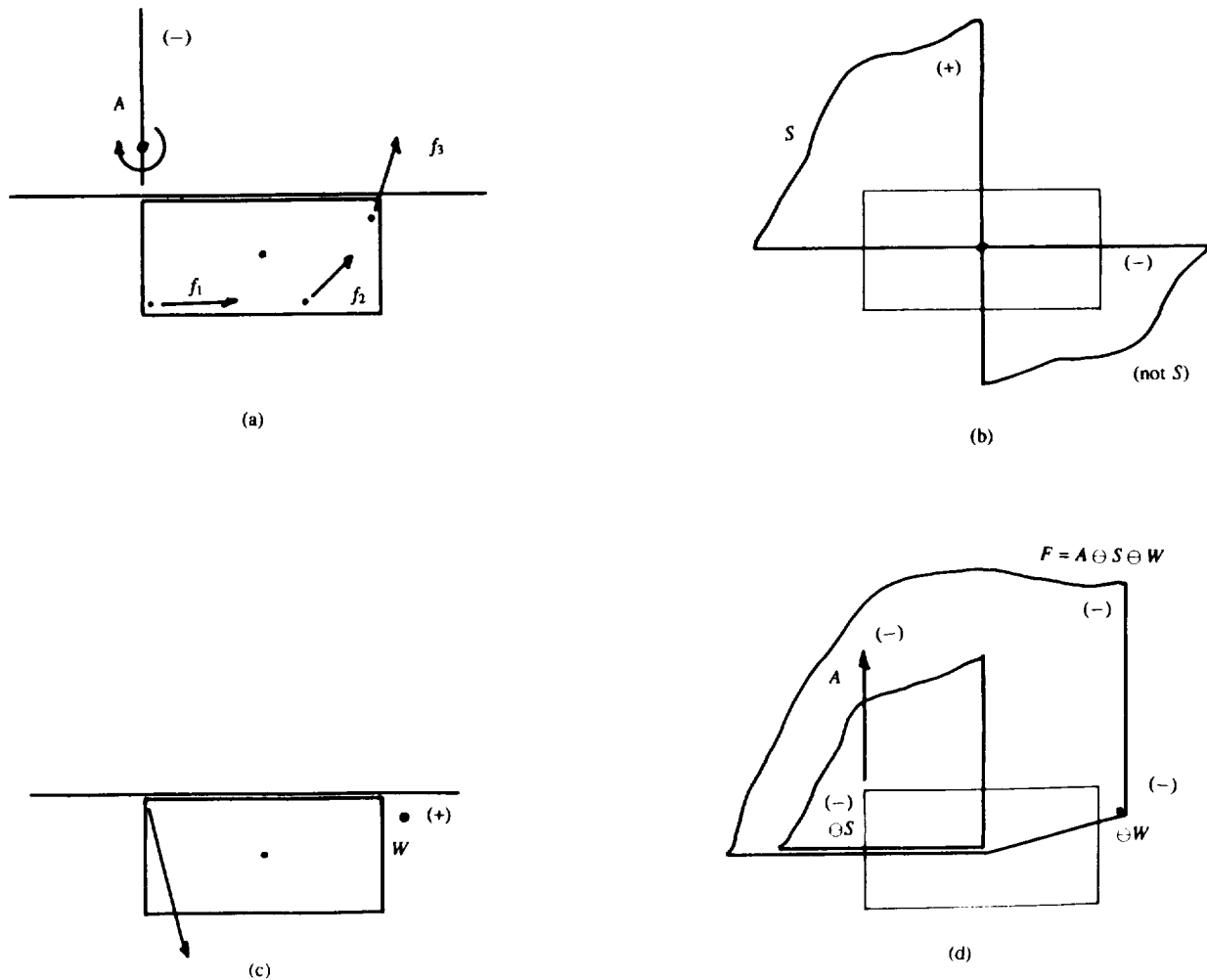


Figure 6: Constructions for a rotating contact mode. The block accelerates to the left, with the right corner losing contact. (a) shows the set of acceleration centers A . Note that because the support distribution is confined to the lower-right quadrant with respect to the acceleration centers, the force direction is confined to the upper-right quadrant. This constraint can be transformed into acceleration center space to obtain figure (b). S corresponds to the (+) half, because of a second constraint: the total force must give a positive moment with respect to the center of mass. (c) shows the wall force W . (d) shows the final set of applied forces F .

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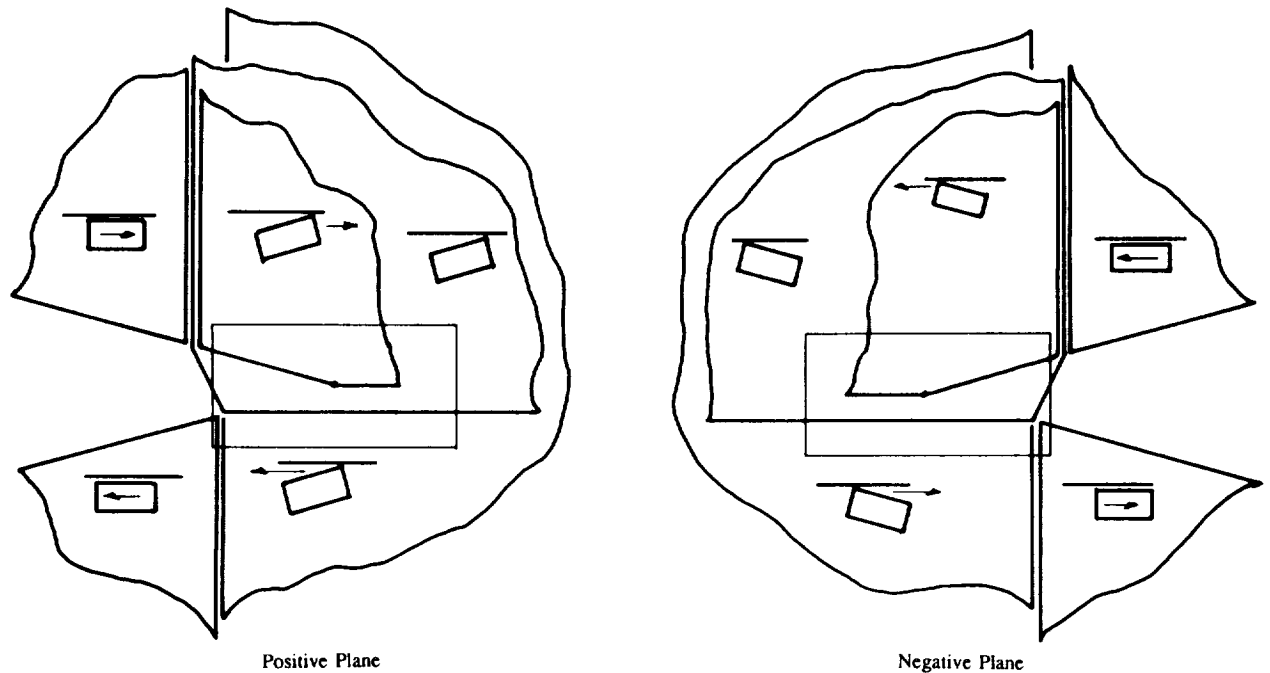


Figure 7: The complete atlas. Not shown is the contact mode that leaves the block motionless, which maps to the entire space of acceleration centers. The gaps in the figure correspond exactly to jamming forces, which fail to move the block no matter how large the force magnitude. Also not shown is the zero line, which in this case is a straightforward continuation of the positive plane.

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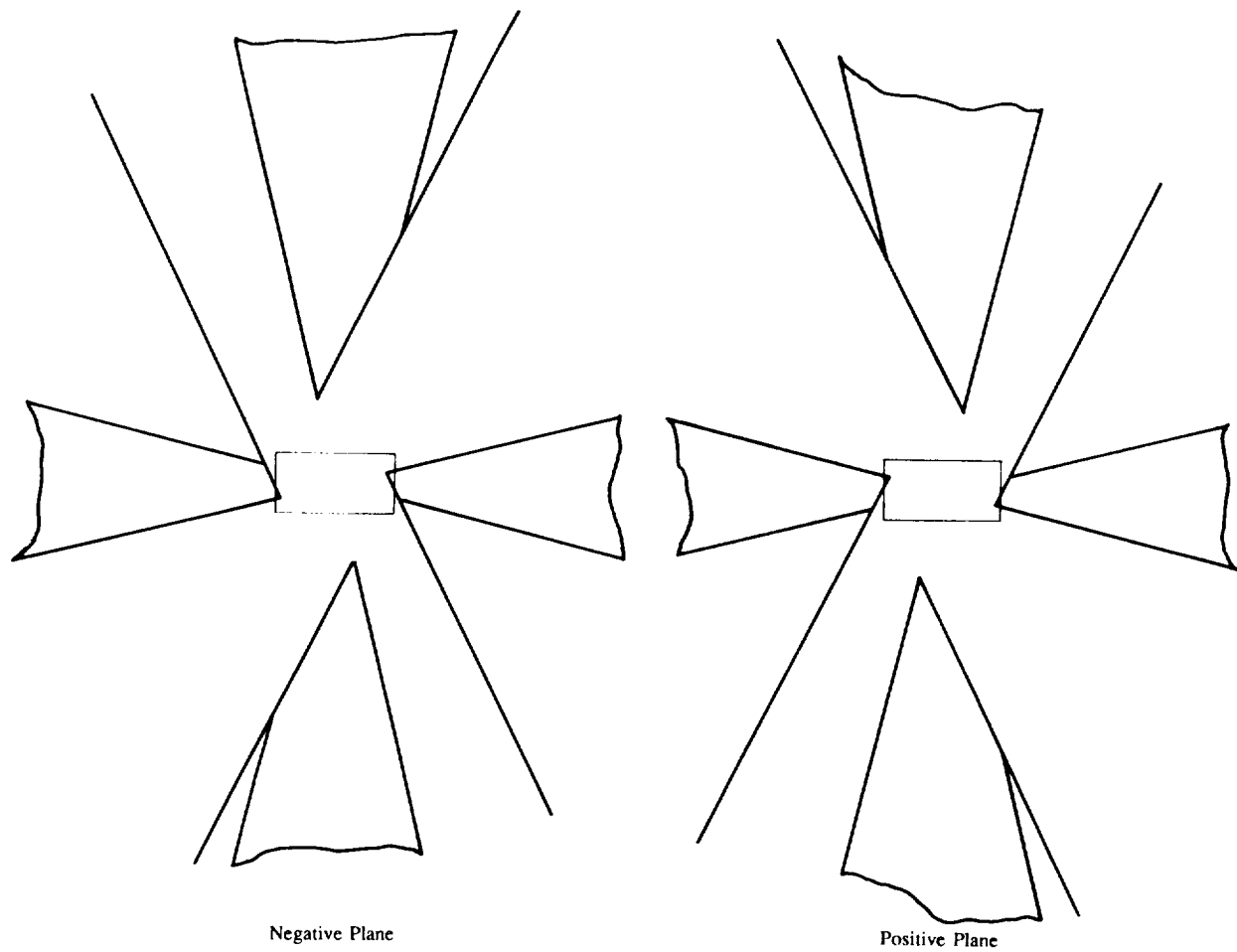


Figure 8: Contact forces arising from a pushing motion.

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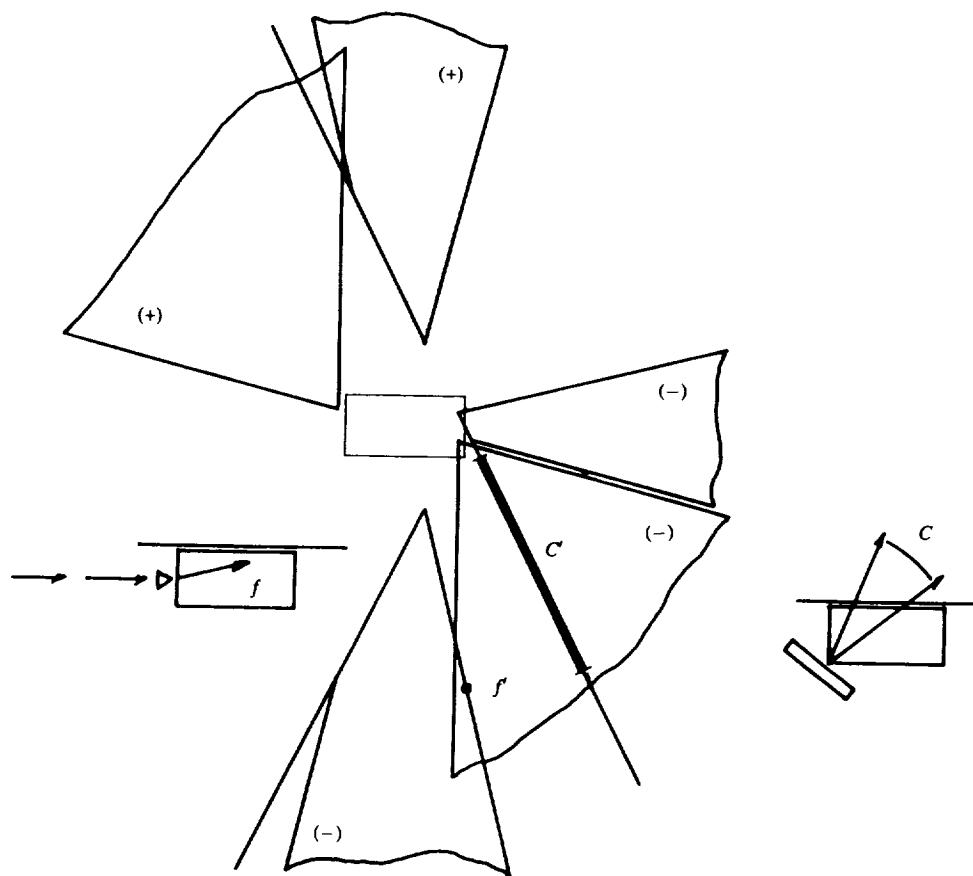


Figure 9: Of the set of possible pushing forces shown in Figure 8, there are just two lobes that intersect the desired forces F of Figure 5. When we intersect the two figures, we obtain two different solutions, illustrated here. f is obtained by pushing on the trailing edge of the block, with a velocity inclined slightly into the wall. C is obtained by pushing on the trailing corner of the block.